

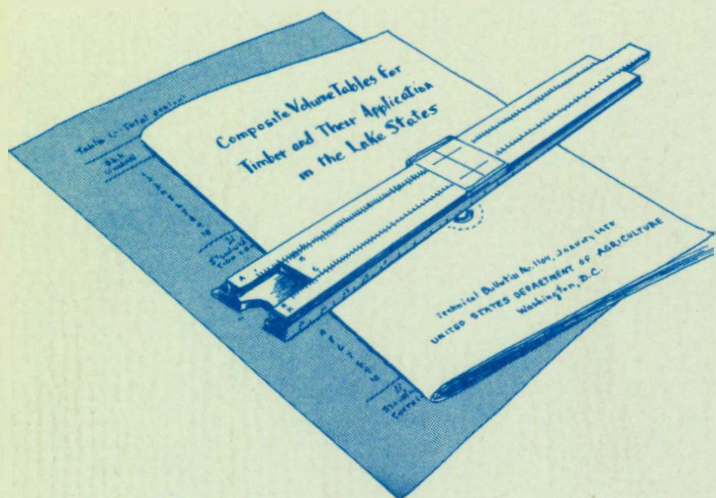
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Adjusting

Shortleaf Pine Volume Tables

for different limits of top utilization

Samuel F. Gingrich



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SAMUEL F. GINGRICH began his Forest Service career in 1957 after six years as an instructor of forest mensuration and management at The Pennsylvania State University. Gingrich received his BS degree from Penn State in 1950 and his MS degree in 1954. He served three years in the Navy Air Corps during World War II. Sam is a member of Xi Sigma Pi and Gamma Sigma Delta, honor societies of Forestry and Agriculture, and the Society of American Foresters. He is responsible for the mensuration aspect of the Station's studies in forest management research. He has authored or co-authored six publications in the field of mensuration.

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Volume tables constructed for a specific minimum top diameter usually must be discarded when the top diameter is changed. To avoid this laborious process, a conversion formula was developed for adjusting any existing shortleaf pine (*Pinus echinata* Mill.) volume table to various minimum top diameters, both fixed and variable. The formula is useful when utilization needs make different top diameters appropriate.

A further refinement is suggested by the fact that, converted to a common merchantability standard, existing shortleaf pine volume tables used in the Central States are remarkably similar. This means that a single basic table convenient for regionwide use is feasible. Appropriate conversion factors would adjust it for different minimum top diameters.

The conversion formula was developed by revising the formula for a total-tree/cubic-foot volume table to exclude top volume to various utilization limits. The tops of 350 felled shortleaf pine trees formed the basis of the top-volume estimates. The felled trees were from stands that had been neither thinned nor pruned. Diameter at 4.5 feet ranged from 5 to 15 inches and height ranged from 30 to 75 feet. Volumes given should not be extended much beyond these limits. All diameters except that at breast height are inside bark.

Form and Length of Top Sections

For volume determinations, the top length and form of each felled tree were measured to 5-inch, 4-inch, and 3-inch top diameters inside bark. The geometric form of the sections between the 5-to-4-inch and 4-to-3-inch diameter levels was found to be generally parabolic; therefore, Smalian's formula was used to compute the volume of these two top sections. Differences in volume between that obtained by using the standard formula for a truncated cone and that obtained by use of Smalian's formula (truncated paraboloid) are small:

Diameter (Inches)	Length (Feet)	Volume	
		Cone (Cu. ft.)	Paraboloid (Cu. ft.)
5 - 4	7	0.77	0.78
4 - 3	6	.40	.41

The form of the top sections of the trees ranged from a cone to a paraboloid, the average being nearest to a paraboloid. Therefore the standard formula for the volume of a paraboloid was used. Thus:

$$V = \frac{1}{2} (A \times L) = 0.0245 L$$

where A is the basal area in square feet, L is the length of the 3-inch-top section, and 0.0245 is one-half the basal area of a stem with a 3-inch base diameter.

Top length of small trees decreases sharply as d.b.h. increases (fig. 1). For trees larger than 10 inches d.b.h. however, lengths for each top section are nearly constant.

As would generally be expected, tall trees of a given d.b.h. had longer tops and hence more top volume than short trees (fig. 2). So in the top-volume equations that are presented later in this report, both d.b.h. and total height are used to estimate top volume.

Top-Volume Equation

Several forms of top-volume equations were investigated. The one with the independent variable $\frac{\text{Total height}}{\text{D.b.h. — top diameter}}$ was judged best, being sufficiently precise and simple (table 1). The variance removed by this regression ranged from 62 to 70 percent. Spurr (9)¹ used the same equation form in deriving 4-inch-top-volume equations with a and b coefficients of 0.19 and 0.025 for red spruce and 0.42 and 0.021 for black spruce.

¹Numbers in parentheses refer to Literature Cited, page 9.

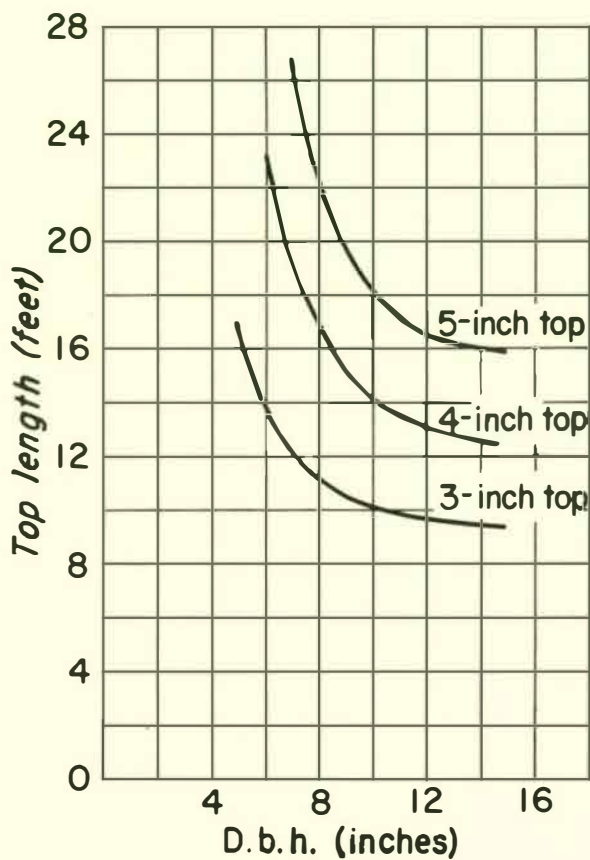
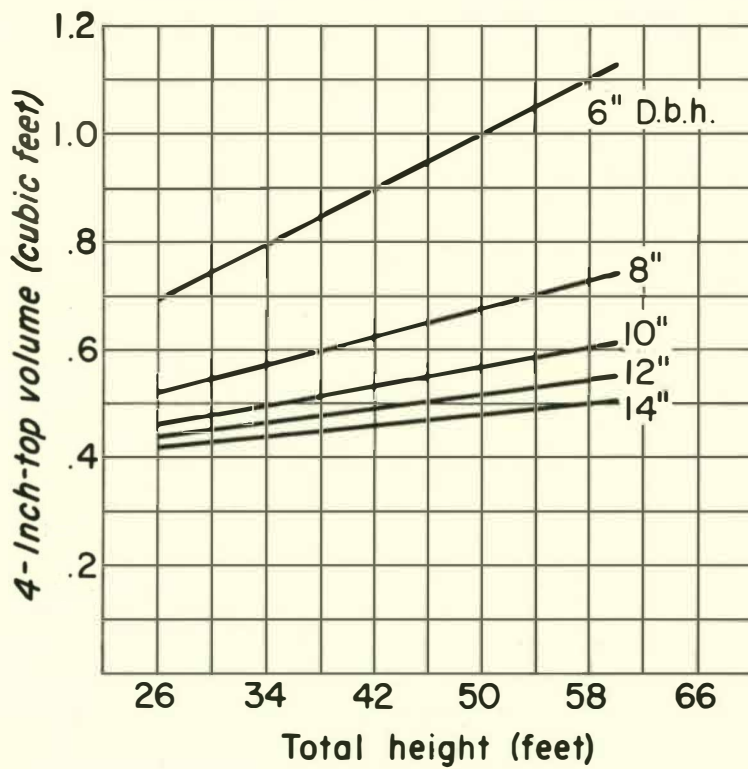


FIGURE 1. — Relation of 3-, 4-, and 5-inch top lengths to d.b.h.

FIGURE 2. — Relation of 4-inch top volume to total height by d.b.h.
 Top volume = $0.354 + 0.0258 \frac{H}{D-4}$



Changing the Merchantability Standards of Cubic-Foot-Volume Tables

Shortleaf pine volume tables for the Central States examined include both published tables (2, 3) and unpublished Forest Service tables. The study indicates that a simple equation estimates the total cubic-foot volume, less bark and stump volume, with a precision of ± 10 percent. The equation is in the form:

$$V = 0.00225 D^2 H$$

where D is the d.b.h. and H is total height in feet. This equation is a modification of the "combined variable" equation

$$V = a + bD^2H$$

where a and b are the regression coefficients.

When stump and bark volume, which are rarely merchantable, were taken out of the total volume equation, a became numerically small and had little effect on the estimation of total volume without stump and bark. The equation is of little practical value since it is neither a total-volume estimate nor a realistic merchantable-volume estimate. But it does provide a convenient base from which to adjust for various merchantability standards that are expressed by merchantable top diameter. This simplified equation can be readily changed to a "form factor"² by converting D^2 to basal area and can be written thus:

$$V = 0.412 BH$$

where B is the basal area at d.b.h. in square feet and H is the total height in feet. Gevorkiantz and Olsen (5) found that a form factor of 0.42 could be used in their composite cubic-foot volume table for the Lake States. Their estimate includes stump volume.

Merchantable cubic-foot volume can be estimated by subtracting the appropriate top-volume equation (table 1) from the general equation. Thus,

$$V_{3'' \text{ top}} = 0.00225 D^2 H - (0.176 + 0.0125 \frac{H}{D-3})$$

Rigorous statistical confirmation is not available for these new merchantable-volume equations, but both field testing and comparison with existing merchantable-volume tables show that the new equations can be used throughout the Central States Region in either plantations or natural stands. Accuracy is usually within ± 5 percent of actual volume. Even under the most extreme conditions of site and tree form, estimates will be within ± 10 percent of actual volume.

²The form factor is commonly used to compare the volume of a tree with the volume of a cylinder of the same diameter and length (height); i.e.

$$\text{Form Factor} = \frac{\text{Volume of tree}}{\text{Volume of cylinder}}$$

TABLE 1. — *Top-volume equations for 3-, 4-, and 5-inch tops, cubic-foot volume, inside bark*

Equation form: $V = a + bx$, where $x = \frac{\text{Total height}}{\text{D.b.h.} - \text{Top diameter}}$

Minimum top diameter (inches)	<i>a</i>	<i>b</i>	Standard error of regression at mean <i>x</i>	Coefficient of Variation	Correlation coefficient <i>r</i>
			<i>Cubic feet</i>	<i>Percent</i>	
3	0.176	0.0125	± 0.054	16	84
4	.354	.0258	± .146	18	81
5	.539	.0462	± .259	19	79

Where reliable volume tables based on local data are available, they should be adjusted in preference to the tables given in the appendix. Any shortleaf pine table for merchantable volume can be adjusted for different top diameters by using the percent tables given in the appendix (tables 7 and 8). Adjusting local merchantable-volume tables for different top diameters does not greatly alter overall accuracy.

Recommended Merchantable-Volume Equations For Fixed Top Diameters

Merchantable cubic-foot volume equations were developed for top diameters of 3, 4, and 5 inches (table 2). The approximate equivalent of these equations is given in the combined-variable form previously mentioned. Volume tables derived from these equations are in the appendix (tables 4, 5, and 6). This transformation results in a simplified equation form that is becoming a standard expression of cubic-foot volume. Recent studies (1, 4, 6, 8) in volume table methodology indicate that the combined-variable equation is superior to other arithmetic equations and nearly as good as the logarithmic volume equation. Good correlations are reported between the combined-variable and the more complex logarithmic expressions of volume. A unique feature of a combined-variable equation is that the volume table generally may be expressed graphically by a single straight line, although sometimes there may be a slight curve in the lower range of D^2H .

Worthy of special mention is the magnitude of the coefficient of the combined-variable equations shown. The *b* coefficients in all three equations are nearly equal and are only slightly larger than the single coefficient of the simplified total-volume equation. Since the *b* coefficient determines the slope of the regression line, a graphical presentation of these equations would result in a series of parallel lines separated on the vertical scale to

the extent of the a coefficients. This simply means that the merchantable-volume equations are related by an additive constant. Differences between merchantable-volume estimates per tree can be obtained by subtracting the appropriate a coefficient. Thus, when utilization standards are changed from a 5- to a 4-inch top the resultant increase in merchantable volume per tree is 0.63 cubic feet (1.28-0.65); from a 4- to 3-inch top, 0.33 cubic feet (0.65-0.32); and from a 5- to 3-inch top, 0.96 cubic feet (1.28-0.32). This feature of merchantable volume was examined in several other volume tables published for coniferous species (7, 10), and in general was confirmed.

TABLE 2. — *Comparison of merchantable top-volume equation with synthesized combined variable equation*

Minimum top diameter (inches)	Volume equation based on top volume	Approximate equivalent combined variable equation
3	$V = 0.00225 D^2 H - (0.176 + 0.0125 \frac{H}{D-3})$	$V = -0.32 + 0.00226 D^2 H$
4	$V = 0.00225 D^2 H - (0.354 + 0.0258 \frac{H}{D-4})$	$V = -0.65 + 0.00226 D^2 H$
5	$V = 0.00225 D^2 H - (0.539 + 0.0462 \frac{H}{D-5})$	$V = -1.28 + 0.00229 D^2 H$

Relation Between Total and Merchantable Height

Total height is generally easier to measure than merchantable height. Reasonably accurate estimates of merchantable height can be obtained once the relation between merchantable and total height is established (table 3). In fact, estimates of merchantable heights, based on total height, are generally more accurate than direct ocular estimates. Trees on good sites had longer tops above specified top diameters than did trees on poor sites but this was because they were taller; the ratio of merchantable height to total height was about the same regardless of site. By using total height in the top-volume equation, most of the variation in top volume due to site was accounted for. This point is made with some reservation due to the lack of site-evaluation data for plantation pine.

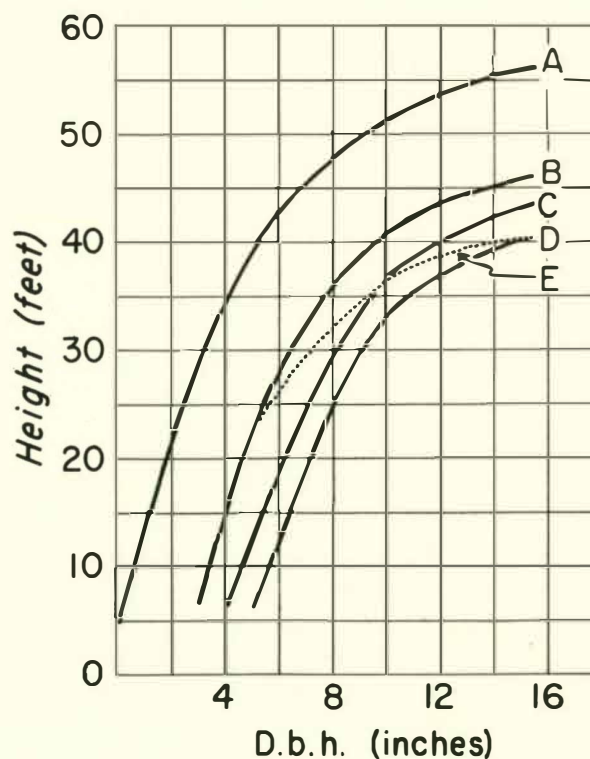
In small trees merchantable height increases as total height increases, but at a faster rate (fig. 3). For example, a tree growing from 6 to 8 inches in d.b.h. increases about 5 feet (42 to 47) in total height. At the same time its merchantable height to a 4-inch top increases about 12 feet (18 to 30). This rapid increase in merchantable volume in small trees is often overlooked in making growth predictions from merchantable-volume tables.

Large volume increases due to merchantable height increase have the same effect on growth predictions as does ingrowth. Growth predictions based on the period of ingrowth and rapid increase in merchantable height may be gross over-estimations unless their factors are considered.

TABLE 3. — *Relation between total and merchantable heights of shortleaf pine: top diameters fixed or variable*

D.b.h. (inches)	Merchantable height as a percent of total height				
	Fixed top diameter, inside bark			Variable top diameter, inside bark	
	3 inches	4 inches	5 inches	Top diameter	
	Percent	Percent	Percent	Inches	Percent
5	59	—	—	3.0	59
6	68	45	—	3.2	63
7	73	56	41	3.4	67
8	77	64	53	3.6	70
9	79	70	60	3.8	72
10	80	72	65	4.0	72
11	81	75	67	4.2	72
12	82	76	69	4.4	72
13	82	77	70	4.6	72
14	83	77	71	4.8	72
15	83	78	72	5.0	72

FIGURE 3. — *Merchantable heights related to total height: (A) Total height, (B) 3-inch fixed top diameter, (C) 4-inch fixed top diameter, (D) 5-inch fixed top diameter, (E) variable top diameter.*



Fixed Versus Variable Top Diameters

It was quite obvious from a study of other top characteristics, such as live-crown position and branching, that using a fixed top diameter imposes stricter merchantability standards as d.b.h. increases. For example, the 3-inch top on a 6-inch tree usually extends below the live crown, while in a 12-inch tree it does not extend to even the base of the live crown. If utilization standards were based on percent of total volume used, cutting a 12-inch tree to a 6-inch top would be better utilization than cutting a 6-inch tree to a 4-inch top.

Top branching and taper characteristics suggest that a variable top diameter offers a more realistic utilization standard than a fixed top diameter, even for small products. The recommended variable top diameters shown (table 3) will result in nonmerchantable tops of the same length (but not volume) for all d.b.h. classes. For trees larger than 8 inches in diameter about 72 percent of the total tree height will be utilized.

The use of variable top diameters generally results in a more realistic estimate of merchantable volume, even where small products are anticipated (table 9). While it may be possible to cut to smaller tops, it is often impractical. Top diameters should be determined not only by minimum product specifications but top characteristics as well.

Literature Cited

- (1) Beers, Thomas W., and Gingrich, Samuel F.
1958. CONSTRUCTION OF CUBIC-FOOT VOLUME TABLES FOR RED OAK IN PENNSYLVANIA. *Jour. Forestry* 56(3): 210-214.
- (2) Boggess, W. R., and Olson, C. E., Jr.
1959. CUBIC-FOOT VOLUME TABLES FOR SHORLEAF PINE PLANTATIONS IN SOUTHERN ILLINOIS. *Univ. of Ill. Agr. Expt. Sta. Forestry Note* 81, 2 pp.
- (3) Clark, F. Bryan, and Williams, Robert D.
1958. CUBIC FOOT VOLUME TABLES FOR PLANTED SHORLEAF PINE. U.S. Dept. Agr., Forest Serv., Cent. States Forest Expt. Sta. Note 121, 2 pp.
- (4) Cooper, R. S., and Olson, D. F., Jr.
1958. VOLUME DETERMINATIONS FOR SECOND-GROWTH SLASH AND LONGLEAF PINE IN NORTHEAST FLORIDA. U.S. Dept. of Agr., Forest Serv., Southeast. Forest Expt. Sta. Paper 92, 11 pp., illus.
- (5) Gevorkiantz, S. R., and Olsen, L. P.
1955. COMPOSITE VOLUME TABLES FOR TIMBER AND THEIR APPLICATION IN THE LAKE STATES. U.S. Dept. Agr. Tech. Bul. 1104, 51 pp.
- (6) Golding, D. L., and Hall, O. F.
1961. TESTS OF PRECISION OF CUBIC-FOOT TREE-VOLUME EQUATIONS ON ASPEN, JACK PINE, AND WHITE SPRUCE. *Forestry Chron.* 37(2): 123-132.
- (7) Haney, G. P., and Kormanik, P. P.
1962. CUBIC-FOOT VOLUME TABLES FOR SHORLEAF PINE IN THE VIRGINIA-CAROLINA PIEDMONT. U.S. Dept. Agr., Forest Serv., Southeast. Forest Expt. Sta. Research Note 170, 2 pp.
- (8) Smith, J. H. G., and Ker, J. W.
1957. TIMBER VOLUME DEPENDS ON D^2H . *Brit. Columbia Lumberman*, Sept. ed., 2 pp.
- (9) Spurr, S. H.
1952. FOREST INVENTORY. The Ronald Press Co., New York, 476 pp.
- (10) U.S. Forest Service
1929. VOLUME, YIELD, AND STAND TABLES FOR SECOND-GROWTH SOUTHERN PINES. U.S. Dept. Agr. Misc. Pub. 50, 202 pp.

Appendix

TABLE 4. — *Merchantable volume (excluding bark) for shortleaf pine to a 3-inch top, inside bark. Stump height = 1/2 d.b.h.*

$$V = -0.32 + 0.00226 D^2H$$

(In cubic feet)

D.b.h. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
5	0.81	1.09	1.38	1.66	1.94	2.22						
6		1.71	2.12	2.53	2.93	3.34	3.75					
7			3.00	3.56	4.11	4.66	5.22	5.77				
8				4.74	5.47	6.19	6.91	7.64	8.36			
9				6.09	7.00	7.92	8.83	9.75	10.66			
10				7.59	8.72	9.85	10.98	12.11	13.24	14.37		
11					10.62	11.99	13.35	14.72	16.09	17.45	18.82	
12					12.70	14.32	15.95	17.58	19.21	20.83	22.46	
13					14.96	16.87	18.78	20.69	22.60	24.51	26.42	
14						19.61	21.83	24.04	26.26	28.47	30.69	32.90
15						22.56	25.10	27.65	30.19	32.73	35.28	37.82

TABLE 5. — *Merchantable volume (excluding bark) for shortleaf pine to a 4-inch top, inside bark. Stump height = 1/2 d.b.h.*

$$V = -0.65 + 0.00226 D^2H$$

(In cubic feet)

D.b.h. (inches)	Total height in feet										
	25	30	35	40	45	50	55	60	65	70	75
6	1.38	1.79	2.20	2.60	3.01	3.42					
7		2.67	3.23	3.78	4.33	4.89	5.44				
8			4.41	5.14	5.86	6.58	7.31	8.03			
9			5.76	6.67	7.59	8.50	9.42	10.33			
10			7.26	8.39	9.52	10.65	11.78	12.91	14.04		
11				10.29	11.66	13.02	14.39	15.76	17.12	18.49	
12				12.37	13.99	15.62	17.25	18.88	20.50	22.13	
13				14.63	16.54	18.45	20.36	22.27	24.18	26.09	
14					19.28	21.50	23.71	25.93	28.14	30.36	32.57
15					22.23	24.78	27.32	29.86	32.40	34.94	37.49

TABLE 6. — *Merchantable volume (excluding bark) for shortleaf pine to a 5-inch top, inside bark. Stump height = 1/2 d.b.h.*

$$V = -1.28 + 0.00229 D^2H$$

(In cubic feet)

D.b.h. (inches)	Total height in feet									
	30	35	40	45	50	55	60	65	70	75
7	2.09	2.65	3.21	3.77	4.33	4.89				
8		3.85	4.58	5.32	6.05	6.78	7.51			
9		5.21	6.14	7.07	7.99	8.92	9.85			
10		6.74	7.88	9.02	10.17	11.32	12.46	13.60		
11			9.80	11.19	12.57	13.96	15.35	16.73	18.12	
12			11.91	13.56	15.21	16.86	18.51	20.15	21.80	
13			14.20	16.14	18.07	20.01	21.94	23.88	25.81	
14				18.92	21.16	23.41	25.65	27.89	30.14	32.38
15				21.91	24.48	27.06	29.64	32.21	34.79	37.36

TABLE 7. — *Additional cubic-foot volume per tree of shortleaf pine when top-diameter utilization is changed from a 4-inch to a 3-inch top, inside bark (In percent)*

D.b.h. (inches)	Total height in feet											Average
	25	30	35	40	45	50	55	60	65	70	75	
6	24	18	15	13	11	10						15
7		12	10	9	8	7	6					9
8			7	6	6	5	5	4				6
9			6	5	4	4	4	3				4
10			5	4	3	3	3	3	2			4
11				3	3	3	2	2	2	2		2
12				3	2	2	2	2	2	1		2
13				2	2	2	2	1	1	1		2
14					2	2	1	1	1	1	1	1
15					1	1	1	1	1	1	1	1

TABLE 8. — *Additional cubic-foot volume per tree of shortleaf pine when top-diameter utilization is changed from a 5-inch to a 4-inch top, inside bark*
(In percent)

D.b.h. (inches)	Total height in feet										Average
	30	35	40	45	50	55	60	65	70	75	
7	28	22	18	15	13	11					18
8		15	12	10	9	8	7				10
9		11	9	7	6	6	5				7
10		8	6	6	5	4	4	3			5
11			5	4	4	3	3	2	2		3
12			4	3	3	2	2	2	2		3
13			3	2	2	2	2	1	1		2
14				2	2	1	1	1	1	1	1
15				1	1	1	1	1	0	0	1

TABLE 9. — *Merchantable volume for shortleaf pine without bark to a variable top diameter inside bark.*
Stump height = $\frac{1}{2}$ d.b.h.

$$V = -0.39 + 0.00222 D^2H$$

(In cubic feet)

Diameter (inches)		Total height in feet											
D.b.h. top	20	25	30	35	40	45	50	55	60	65	70	75	
5	3.0	0.72	1.00	1.28	1.55	1.83	2.11						
6	3.2		1.61	2.01	2.41	2.81	3.21	3.61					
7	3.4			2.87	3.42	3.96	4.51	5.05					
8	3.6				4.58	5.29	6.00	6.71	7.42				
9	3.8				5.90	6.80	7.70	8.60	9.50	10.40			
10	4.0				7.38	8.49	9.60	10.71	11.82	12.93	14.04		
11	4.2					10.35	11.70	13.04	14.38	15.73	17.07	18.41	
12	4.4					12.40	14.00	15.59	17.19	18.79	20.39	21.99	
13	4.6					14.62	16.49	18.37	20.24	22.12	24.00	25.87	
14	4.8						19.19	21.37	23.54	25.72	27.89	30.07	32.24
15	5.0						22.09	24.58	27.08	29.58	32.08	34.58	37.07

Gingrich, Samuel F.

1962. ADJUSTING SHORTLEAF PINE VOLUME TABLES FOR
DIFFERENT LIMITS OF TOP UTILIZATION. U. S. Dept.
Agr., Forest Serv., Cent. States Forest Expt. Sta. Tech.
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Presents a formula for adjusting shortleaf pine volume tables
used in the Central States to various minimum top diameters.

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